# MATH-427 Set Theory

**Credit Hours:** 3-0

**Prerequisite:** None

**Course Objectives:** Everything mathematicians do can be reduced to statements about sets, equality and membership which are basics of set theory. This course introduces these basic concepts. The course aims at familiarizing the students with cardinals, relations and fundamentals of propositional and predicate logics.

**Core Contents:** Sets, ordered pairs, Arbitrary union and intersections, Generalized De-Morgan's law, direct and inverse image of functions, Relation on sets, equivalence relations, partially ordered relations, lexicographical ordered, Quotient of Equivalence relation# Conditionally completeness, Well-ordered Classes, Axioms of Choice and their application, Hausdorff's Maximal Principle, Zorn's Lemma, Countability axioms, Ordinality axioms

**Course Contents:** Building Sentences, Building Classes, Algebra of Classes, Ordered pairs, Cartesian products, Classes of Ordered pairs, Index Classes, Arbitrary union and intersection and related results, Generalized De-Morgan's laws, Power sets, Basic properties of Function, Surjective, injective and Bijective functions, Composite functions, invertible functions, direct and inverse image of functions, Finite and arbitrary product of family of classes  Projection mappings, Relations on Sets, Equivalence Relations and Order Relations, Partition of a set, Pre-image, restriction and quotient of equivalence relations, Partially ordered and Lexicographical ordered Relations, Comparable sets, Order preserving functions and isomorphisms, Maximal, minimal, maximum, minimum elements, upper bound, lower bound, infimum, supremum of a set and their properties, Conditionally Completeness, Lattice, Boolean Algebra, Well-ordered Classes, Sections, Principle of Transfinite Induction, Axioms of Choice and their applications, Hausdorff's Maximal Principle, Zorn's Lemma, Well-ordering theorems and Conclusions, Axioms of Cardinality and their properties, Special properties of infinite cardinal numbers, Axioms of ordinal numbers and its properties

**Course Outcomes:** Upon completion of this course, the student should be able to understand:

- Arbitrary Union and intersection and Generalized De-Morgan's law, Invertible functions, Direct and inverse image of functions and their properties
- Equivalence, partially ordered relation and Lexicographical ordered relation, partition of a set
- Lattice, infimum, supremum, maximal and minimal elements, Boolean Algebra, conditionally completeness
- Axioms of Choice and their applications, Hausdorff's Maximal Principle, Zorn's Lemma

**Text Book:** C. C. Pinter, "A Book of Set Theory", Dover Publication, New York, Revised Edition (2014).

**Reference Books:**

1. A. A. Fraenkel, "Abstract Set Theory", North Holland Publishing Company, (1966).
2. P. T. Johnstone, "Notes on Logics and Set Theory", Cambridge University Press, (1996)
3. R. R. Stoll, "Set Theory and Logic", Dover Publication Inc., New York, (1979)

| **Weekly Breakdown** | | |
|---|---|---|
| **Week** | **Section** | **Topics** |
| 1 | Chap.1 Sec. 1-3 | Building Sentences, Building Classes and related theorems, Algebra of Classes |
| 2 | Chap.1 Sec. 4-5 | Ordered and unordered pairs, Cartesian products, Classes of Ordered pairs and related theorems |
| 3 | Chap. 1 Sec. 6-7 | Index Classes, Arbitrary union and intersection and related results, Generalized De-Morgan's laws, Sets, Power sets and related theorems |
| 4 | Chap. 2 Sec. 2-4 | Functions, Basic properties of Function, Surjective, injective and Bijective functions, Composite functions, invertible functions and related results, direct and inverse image of functions and theorems |

| | | |
|---|---|---|
| 5 | Chap. 2 Sec. 5-6, Chap. 3 Sec. 1 | Finite Product of family of classes, arbitrary product of family of classes and related results, Projection mappings, The axioms of Replacements and theorems, Relations on Sets |
| 6 | Chap. 3 Sec. 2-4, | Equivalence Relations and Order Relations, Partition of a set and related results, Pre-image, restriction and quotient of equivalence relations |
| 7 | Chap. 4 Sec. 1-2, | Partially ordered and Lexicographical ordered Relations, Comparable sets, Segments of a set, Order preserving functions and isomorphisms, and related results |
| 8 | Chap. 4 Sec. 3-4, | Maximal, minimal, maximum, minimum elements, upper bound, lower bound, infimum, supremum of a set and their properties, Conditionally Completeness, Lattice, Sublattice, Boolean Algebra |
| 9 | **Mid Semester Exam** | |
| 10 | Chap. 4 Sec. 5, Chap. 5 Sec. 1-2 | Well-ordered Classes, immediate predecessor, Sections, Principle of Transfinite Induction, Axioms of Choice and their proofs |
| 11 | Chap. 5 Sec. 3-4, | Application of Axioms of Choice, Hausdorff's Maximal Principle, Zorn's Lemma |
| 12 | Chap. 5 Sec. 5-6, | Well-ordering theorems, Conclusions, and Examples |
| 13 | Chap. 7 Sect. 1-3, | Finite and infinite sets, Equipotent of Sets, denumerable set, Properties of infinite sets, and countability axioms |
| 14 | Chap. 8 Sec. 1-2, 4 | Axioms of Cardinality, Operations on Cardinal numbers, Special properties of infinite cardinal numbers |
| 15 | Chap. 9 Sec. 1-2 | Axioms of Ordinal numbers, Operations on ordinal numbers and related results |
| 16 | Chap. 9 sec. 3 | Well-orderedness of ordinal numbers, ordering of ordinal numbers and its properties |
| 17 | | Review |
| 18 | **End Semester Exam** | |